Strategies for B-Str8ts puzzles

Version 1.1, Date 20/01/2024

by BP

Introduction

This text describes strategies to solve Str8ts puzzles of the B variant. It was very much inspired by and relies on „Str8ts strategies“ by SlowThinker: https://de.slideshare.net/SlowThinker/Str8ts-basic-and-advanced-strategies.

I hope you‘ll find this text helpful. If you have comments, corrections or criticism please mail to BP.Str8ts@web.de.

This text is dedicated to Andrew, who provides https://www.Str8ts.com/weekly_Str8ts.aspx.

And to morl who presented the first B-Str8ts puzzle on that same site.

And to Leren who provided the next couple of hundreds on that same site.

And to kst who took up the job when Leren quit.

You all have given me and my partner countless hours of pleasure. Thank you!

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Definition B-Str8ts puzzle

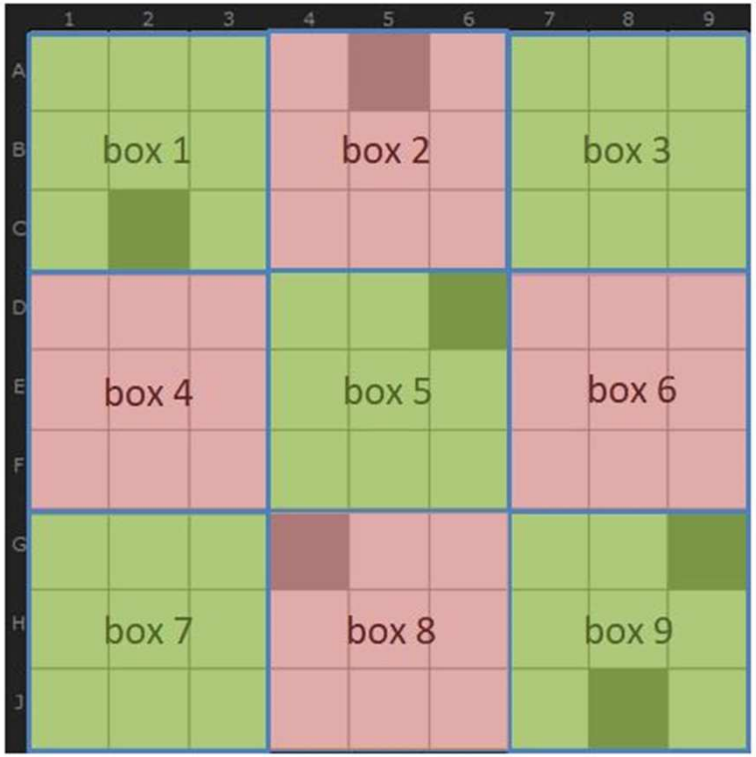
To my knowledge the B variant for Str8ts puzzles was invented by morl and first presented on https://www.Str8ts.com/weekly_Str8ts.aspx.

To B-Str8ts puzzles the same rules apply as to normal Str8ts puzzles plus one additional rule:

The 9x9-grid of cells is divided into nine 3x3-subgrids, called boxes. For each of these boxes the following condition is set: Each digit from 1 … 9 may appear **at most once** in the box.
Thus a B-Str8ts puzzle can be viewed as a Str8ts puzzle with additional Sudoku rules.

Note that the digits within a box need not form a straight, i.e. they need not be consecutive.

Note that the boxes are enumerated box 1 to box 9 from upper left to lower right.



Strategies – Overview

The first thing to know about solving a B-Str8ts puzzle is that you can use all known strategies for normal Str8ts puzzles, e.g. wings, fishes, naked/hidden pairs, settis, ... for B-Str8ts puzzle as well. With one exception: The Unique Solution Constraint (UR for short). In B-Str8ts puzzles UR arguments only work under certain circumstances, as I will show later.

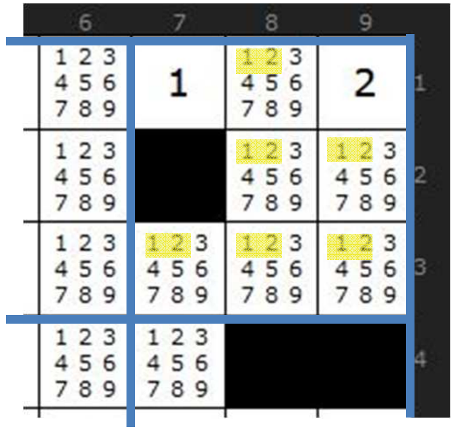
However, thanks to the box condition, there are even more strategies you can use. They fall into two categories: Candidate Elimination and B-Setti's Rules. While candidate elimination works quite straightforwardly, B-Setti's rules are more complex and much more powerful than the normal Setti's rule.

Note: In the remainder of this text I will give some hints for strategies for eliminating candidates in B-Str8ts puzzles and then explain in detail the B-Setti's rules. Before reading on make sure you know about basic Str8ts terminology as described in „Str8ts strategies" by SlowThinker:

https://de.slideshare.net/SlowThinker/Str8ts-basic-and-advanced-strategies.

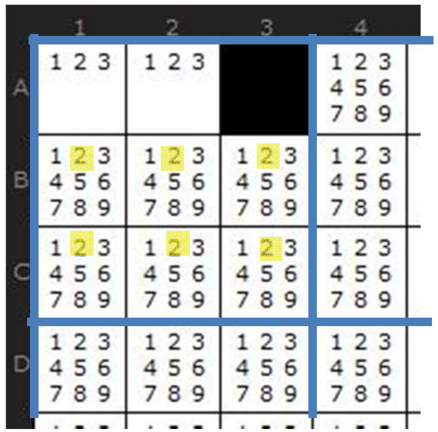
Eliminating candidates – Easy observations

Let‘s start with some obvious observations and simple examples:



First and trivial **observation**: If a digit is already fixed in a box, it can be deleted from all other cells of this box.

Example 1: A7=1, A9=2 fixed in box 3 => 1 and 2 can be removed from all other cells of box 3.



This also works for sure candidates:

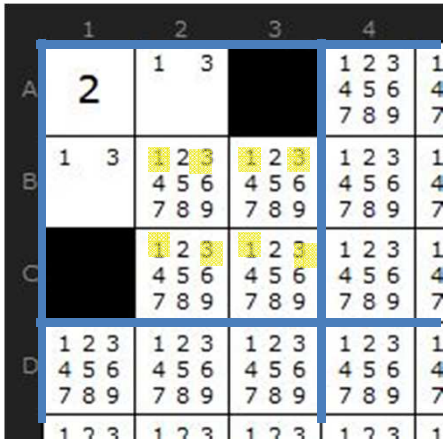
Example 2: 2 is a sure candidate in A12 => 2 can be removed from all other cells of box 1

Eliminating candidates – Naked pairs/triples/…

If two digits form a naked pair in a box they can be deleted from all other cells of this box.

Example 3: A2 = B1 = 13 => 1 and 3 can be removed from all other cells of box 1.

This also works for naked triples / quadruples / quintuples / …

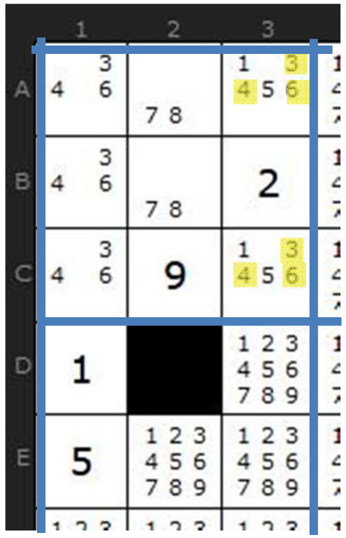


Eliminating candidates – Singles / Hidden pairs/triples/…

Singles: If a digit must be in a box (i.e. it is a sure candidate in the box) and there is just one cell left for it, you can assign the digit to this cell.

Hidden pairs: If two digits are sure candidates in a box and can only be in the same two cells, all other candidates can be deleted from these cells.

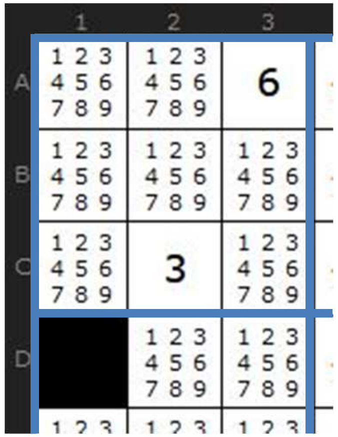
Example 4: Box 1 contains all nine digits, therefore it contains 1 and 5. Since 1 and 5 are only possible in A3 and C3 we can delete all other candidates (3,4,6) from these cells.



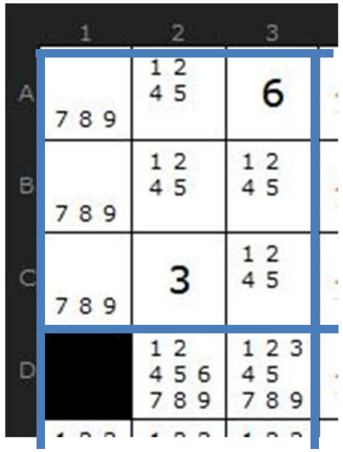
This also works for hidden triples / quadruples / quintuples / …

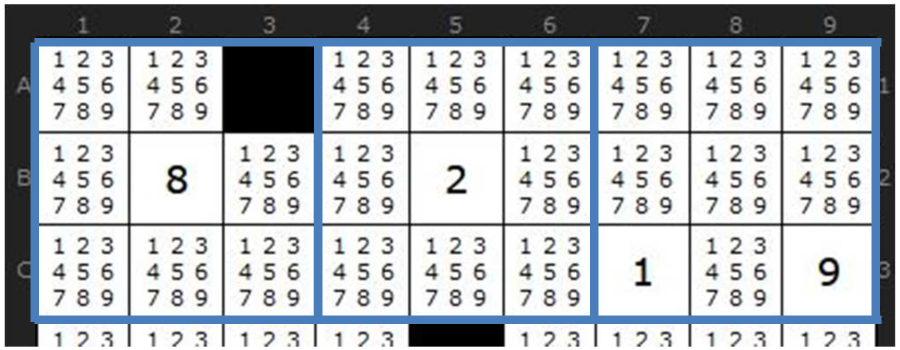
Eliminating candidates – Short compartments within a box

If you tackle a new B-Str8ts puzzle and do not know where to start it's often a good idea to have a look at short compartments of length 2 or 3 that lie **completely within a box**. If there are digits already fixed in this box they often give valuable clues for the short compartment.



Example 5:
Look at compartment ABC1:
3,6 not in ABC1 => ABC1=789.
Note: In consequence ABC1=789 is a naked triple.



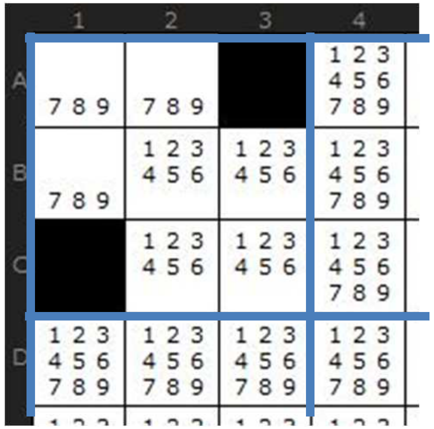


Example 6:
Look at compartment A12:
8 not in A12 => A12 low, i.e. A12=123 and 2 sure in A12.
Note: In consequence A8=2 is single in C.

Eliminating candidates – Intersecting compartments within a box

If you have already looked at all short compartments you might want to look at situations where **two short compartments that lie (mostly) within the same box intersect**. Checking the options for the cell in which the compartments intersect often leads to good results.

Example 7: Look at the two compartments A12 and AB1 in box 1 that intersect in A1 and form a naked triple A12,B1=789. Check possible options for A1:
A1=7 => A2=B1=8. Contradiction.
A1=9 => A2=B1=8. Contradiction.
A1=8, A2=B1=79.
Therefore A1=8.

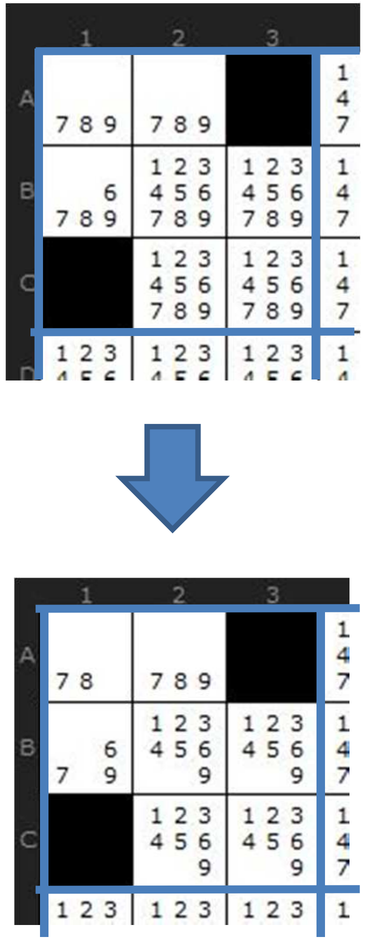


Another way of looking at it:
8 is a sure candidate in A12, therefore not in B1.
8 is a sure candidate in AB1, therefore not in A2.
8 can only be in A1.

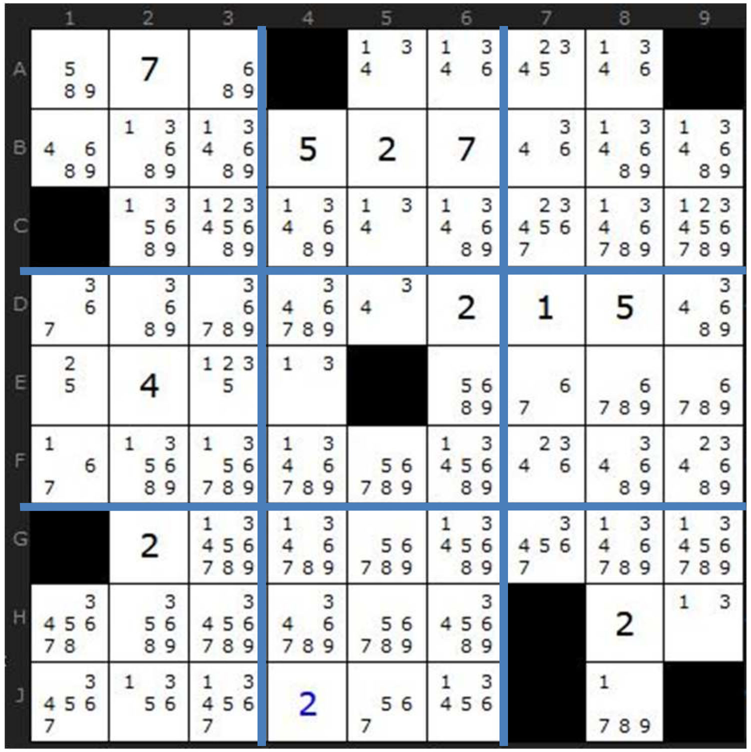
Eliminating candidates – Intersecting compartments within a box

Example 8: Look at compartments A12 and AB1 in box 1:
8 in A12 => 8 not in B1 => 9 not in A1.
Check the other options for A1:
A1=7 => A2=8, B1=6.
A1=8 => A2,B1=79.
Therefore there are three valid solutions:
B2/A1/A2 = 6/7/8 or 7/8/9 or 9/8/7.

Note that the digits in every solution are **consecutive**. Note also that 7,8 belong to each solution. We knew already that 8 is a sure candidate in A12. We know now that 7 is a sure candidate in A12,B1 and can therefore be removed from all other cells of box 1.



Eliminating candidates – Intersecting compartments within a box



Example 9: is.gd/kst_680_B (*) after basic eliminations

Look at A123 and AB1 in box 1:
A1=5 => A3=6, B1=4
A1=8 => B1=9, A3=6
A1=9=> B1=A3=8. Contradiction.
Therefore: A1=58, B1=49, A3=6.

Look at DEF1 and E1234 (mostly) in box 4:
E1=2 => D1=3, F1=1, E3=5, E4=3.
E1=5 => D1=F1=67, E3=2, E4=3.
Therefore: E1=E3=25, E4=3.
Now it‘s easy to move forward with D5=4, …

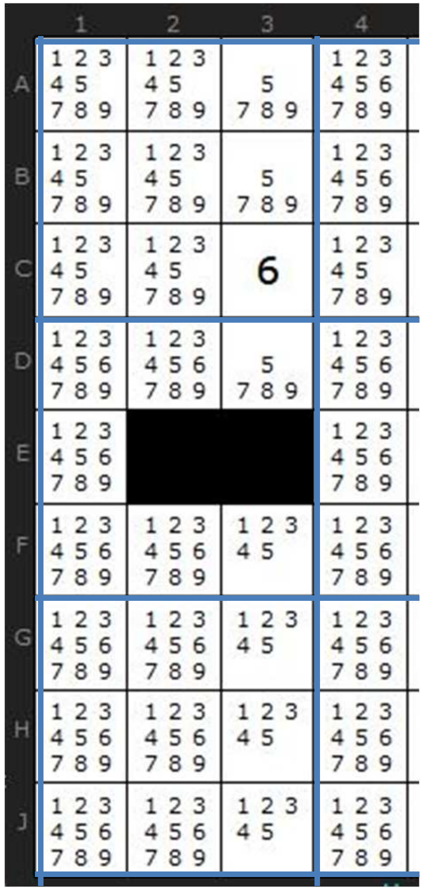
Note: This kind of carefully checking for options is often called **careful elimination (CE).**

(*) Posted by kst in week #680 on https://www.Str8ts.com/weekly_Str8ts.aspx.

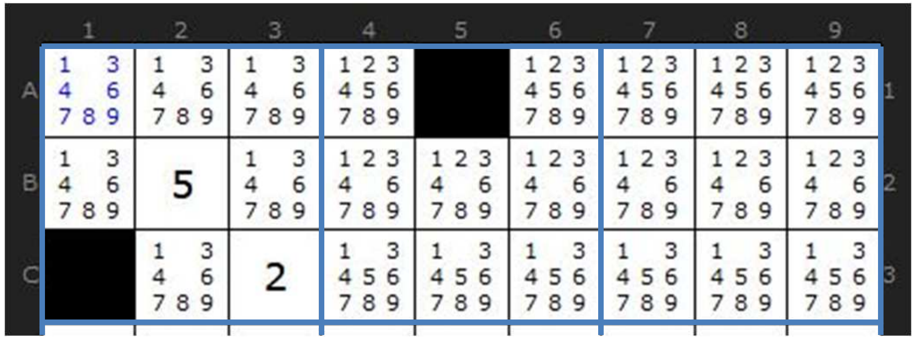
Eliminating candidates – High/low decisions

The box rules are often very helpful to decide high/low questions in rows and columns.

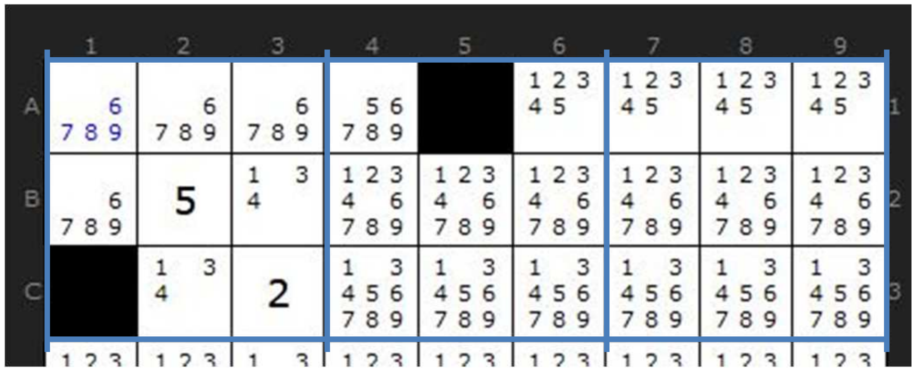
Example 10: High/low in column 2:
ABCD3 is high. If ABCD2 is high as well then all six cells in ABC2,ABC3 have to be >=5, which is not possible.
Therefore ABCD2 is low.



Eliminating candidates – High/low decisions

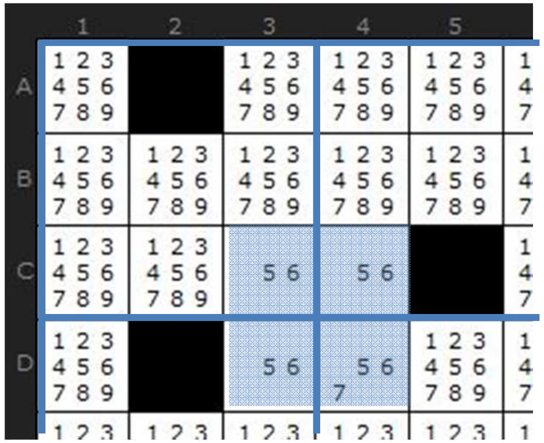


Example 11: High/low in row A:
Look at box 1: Because of B2=5 the following is true:
If A1234 high then A123>5 and B1>5. This gives a **naked quadruple** A123,B1=6789.
If A1234 low then A123<5 and B1<5. Since C3=2 this is not possible.
Therefore A1234 is high.



UR – Unique solution constraint

The unique solution constraint (UR for short) makes use of the fact that a properly designed (B-)Str8ts puzzle has exactly one solution. As said using an UR argument in B-Str8ts puzzle is tricky as the following **Example 12** shows:

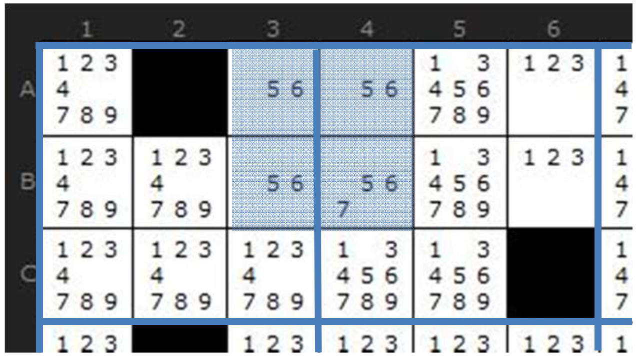


If this were a „normal“ Str8ts puzzle we could use an UR argument to deduce that D4=7, because: If C34=D34=56 then for any valid solution we could construct another valid solution by swapping the positions of 5 and 6 in CD34.

However in this B-puzzle swapping the positions of 5 and 6 in CD34 would alter the set of digits within the affected boxes and thereby probably violate the box rules! For example: If we had C3=D4=5, C4=D3=6 swapping would remove a 5 from box 1 and box 5 respectively and add a 6 instead. It would also remove a 6 from box 2 and box 4 respectively and add a 5 instead.

UR – Unique solution constraint

If you want to use an UR argument in a B-Str8ts puzzle make sure you do not violate the box rules!



In **Example 13** we can use an UR argument to deduce that B4=7, because: If AB34=56 then for any valid solution we can construct another **valid** solution by swapping 5 and 6 in AB34, because this swap will **not** change the set of digits within boxes 1 and 2, just the position of 5 and 6 within these boxes.

B-Setti‘s rules

The strategies explained so far can be very helpful in eliminating candidates from cells, but the real fun in doing B-Str8ts puzzle lies in using B-setti power! B-settis can be a really powerful tool. To explain this let‘s recall Setti‘s rule for „normal“ Str8ts puzzles:

Setti‘s rule: In the final solution of a Str8ts puzzle each digit occurs in the same number of columns and rows.

This also is true for B-Str8ts puzzles. But for B-Str8ts puzzles we can prove even more:

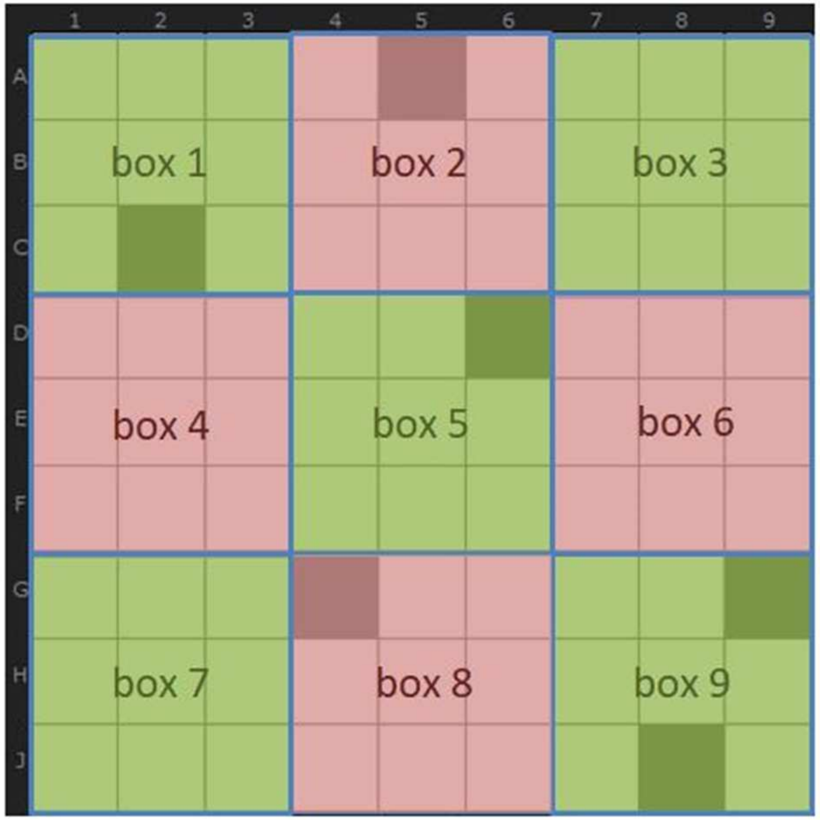
B-Setti‘s rule I: In the final solution of a B-Str8ts puzzle each digit occurs in the same number of columns and rows **and boxes**.

B-Setti‘s rule II: If a digit is missing from a row/column it has to be missing from at least one of the boxes intersecting this row/column.

B-Setti‘s rule III: If a digit is missing from a box it has to be missing from at least one of the rows and at least one of the columns intersecting this box.

B-Setti‘s rules – Easy examples

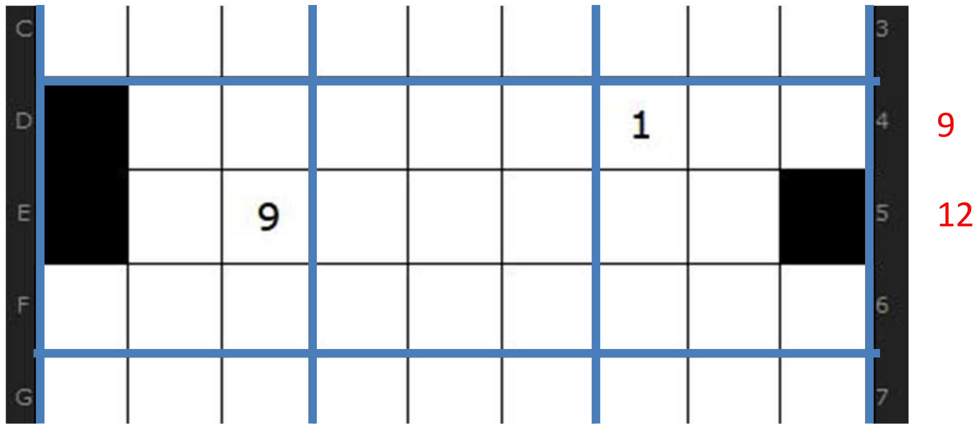
These rules follow easily from the box condition. A proof can be found at the end of this text. Let‘s continue with some trivial **examples 14-16:**



- If a digit is missing from row D then by **B-Setti‘s rule II** it must be missing from at least one of the boxes 4,5,6.
- If a digit is missing from column 2 then by **B-Setti‘s rule II** it must be missing from at least one of the boxes 1,4,7.
- If a digit is missing from box 9 then by **B-Setti‘s rule III** it must be missing from at least one of the rows G,H,J **and** from at least one of the columns 7,8,9.

B-Setti‘s rules – Easy examples

A more concrete example 17:



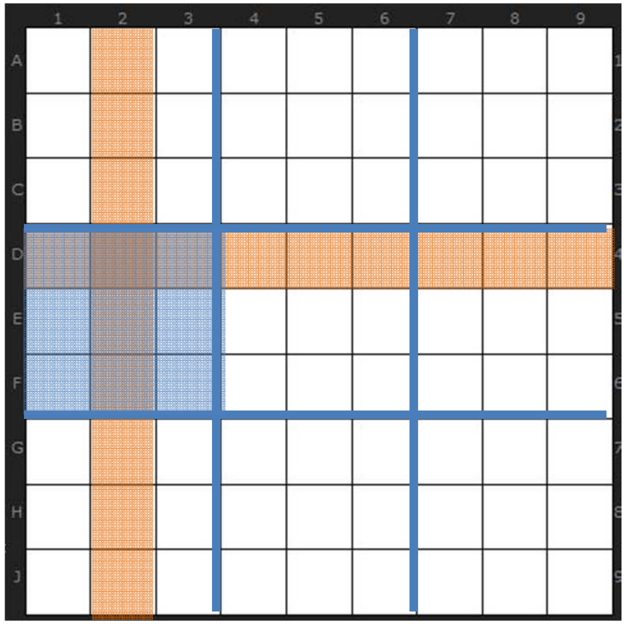
In this example 9 is missing from D, 1 and 2 are missing from E.
B-Setti‘s rule II implies:
Each of 1,2,9 must be missing from one of the boxes 4,5,6. But from which?
Look at 1: 1 is clearly in boxes 5 and 6 therefore it must be missing from box 4.
Look at 9: 9 is clearly in boxes 4 and 5 therefore it must be missing from box 6.
Look at 2: At first glance it‘s not clear whether 2 is missing from box 4 or box 6.
But box 6 is missing just one digit and it‘s missing 9 already. Therefore 2 is missing from box 4.
Now **B-Setti‘s rule III** implies:
1 and 2 must be missing from at least one of columns 1,2,3.
9 must be missing from at least one of columns 7,8,9.

B-Setti‘s rules – Observations

So far so easy. To see how useful B-Setti-power really is we will look at three more complex examples. But before let‘s make some observations that follow directly from the B-Setti‘s rules:

Observation I: If a digit x is missing from just one row and one column it must be missing from the box that intersects that row **and** that column.

Example 18: If 5 is missing from row D and column 2 and is present in all other rows and columns, 5 is missing from box 4 and present in all other boxes.

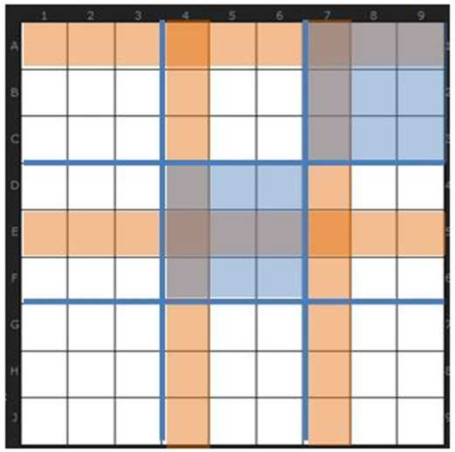


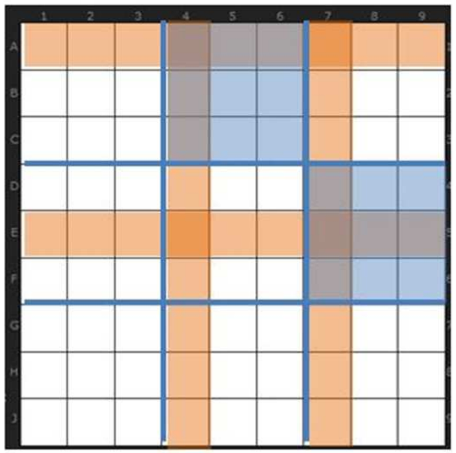
B-Setti‘s rules – Observations

Observation I can be generalized to digits that are missing in two or even more rows/columns. However: To write down this observation in a mathematically correct way would be rather tedious and not very illuminating, therefore I give you two examples instead.

Example 19: If 3 is missing from rows A,E and columns 4,7 and is present in all other rows and columns then it‘s missing in exactly two boxes. These boxes are:

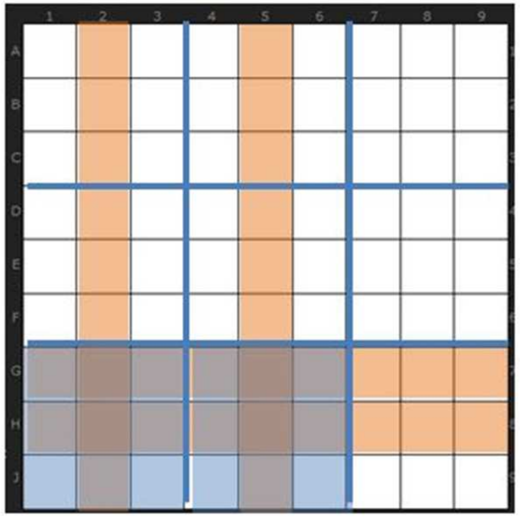
- Either box 3 (which intersects row A and col 7) and box 5 (which intersects row E and col 4).
- Or box 2 (which intersects row A and col 4) and box 6 (which intersects row E and col 7)





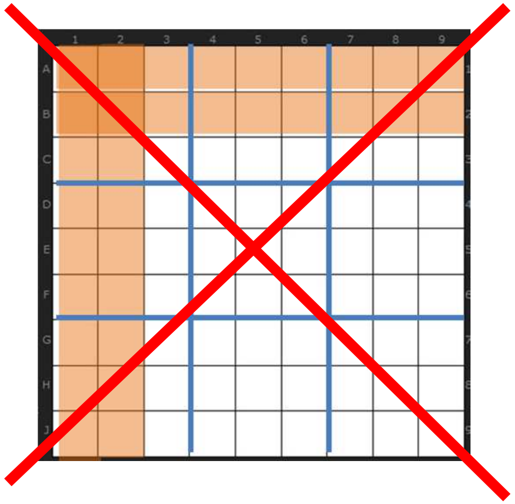
B-Setti‘s rules – Observations

Example 20: If 8 is missing from rows G,H and columns 2,5 and is present in all other rows and columns then it‘s missing in exactly two boxes. These boxes are box 7 and box 8.



B-Setti‘s rules – Observations

Non-Example 21: Note that not all combinations of rows/columns are possible. For example it is not possible that a digit x is missing from rows A,B and columns 1,2 and is present in all other rows and columns, because:
By B-Setti‘s rule I x must be missing in exactly two boxes.
By B-Setti‘s rule III x can only be missing from box 1. Contradiction.

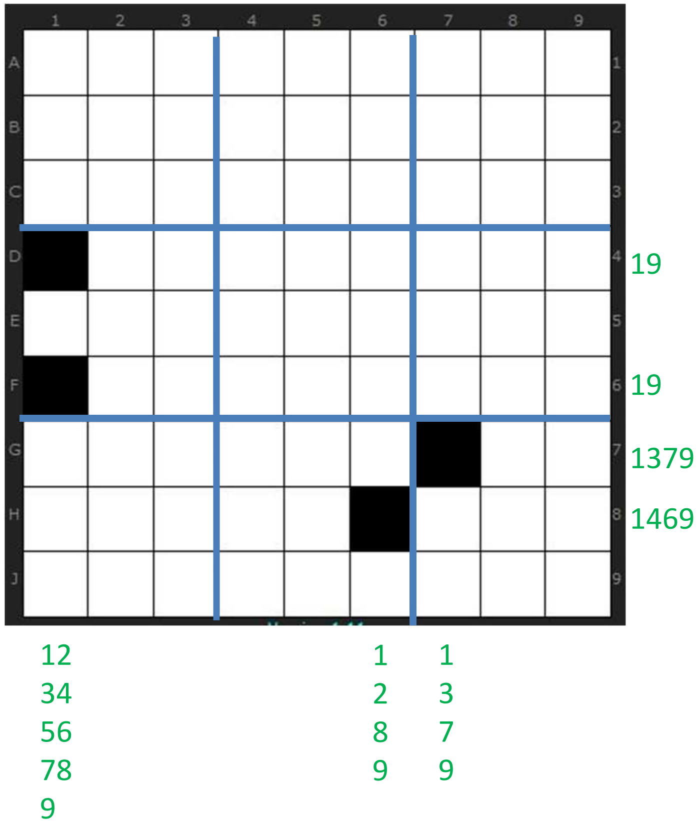


Note: I hope it‘s obvious how these examples can be transformed to digits missing from other pairs of rows/columns or even more than two rows/columns.
Basically you need to keep in mind: The only boxes that might miss a digit x are the boxes where the rows and columns missing x intersect. If there are several options the boxes must be chosen in such a way that each row and column missing x is intersecting with such a box.

B-Setti‘s rules – Advanced Examples

And now some examples to demonstrate the power of B-Setti‘s rules:

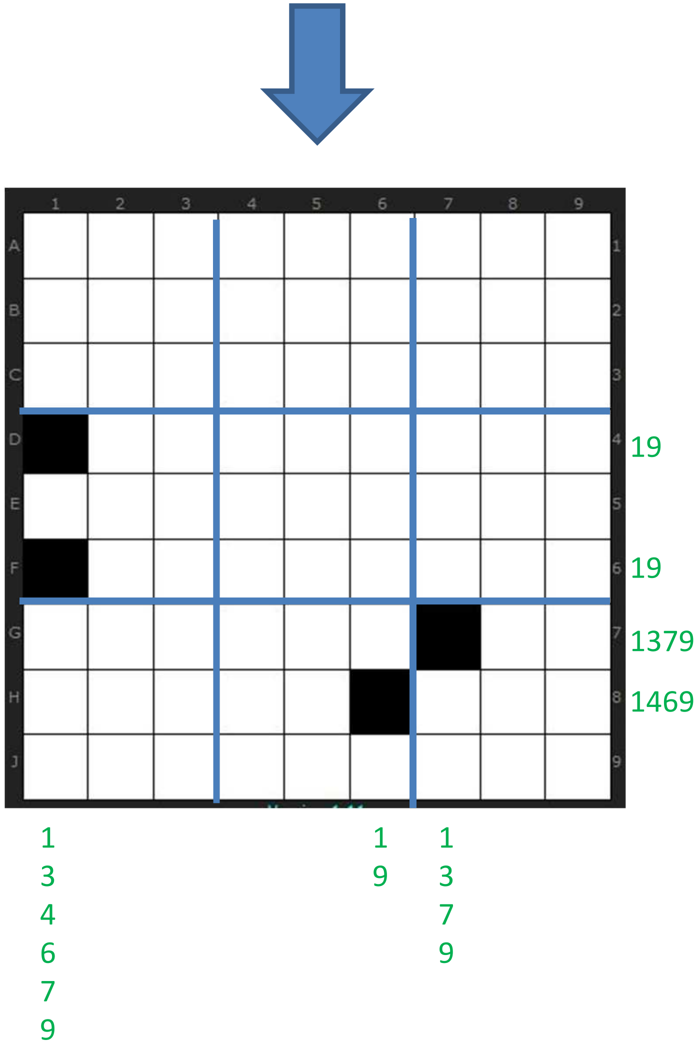
Example 22: (Note that this is not a valid puzzle, since it does not even contain clues. It‘s useful nonetheless, since this example is all about the black cells.)



We start with a black cell analysis (BCA). This means that we analyze the positions of all black cells and then, after each row and below each column, note all digits that **must** be missing from this row/column in red and that **might** be missing in green.

Now by Setti‘s rule for „normal“ puzzles it‘s clear that: Since 2,5,8 belong to all rows these digits belong to all columns. Therefore the corresponding green digits can be removed and we get:

B-Setti‘s rules – Advanced Examples



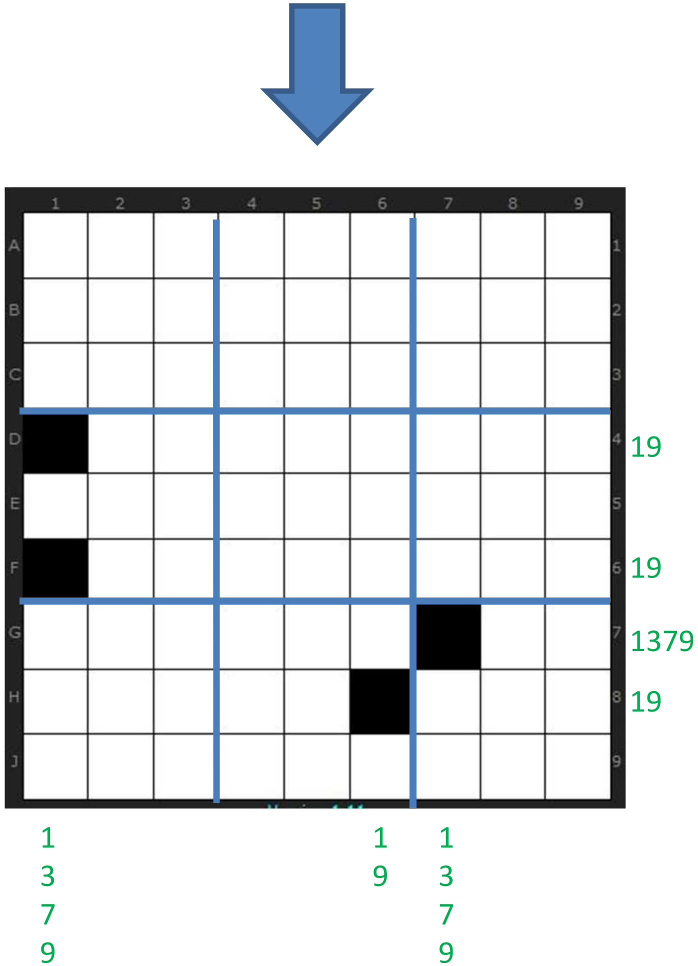
This is as far as we get with the old Setti‘s rule. Now we apply B-Setti‘s rules:

Look at 4: 4 might be missing in row H and col 1. If it is missing in both then by **rule I** it‘s also missing in exactly one box: box 7, where row H and col 1 intersect. But box 7 contains all digits, including 4. Therefore 4 cannot be missing at all!

Look at 6: By the same argument 6 cannot be missing at all!

Note: This is one way of getting rid of 4 and 6 as possible missing digits. We could also have done it by the next step. Quite often there are many different ways to solve a specific B-Str8ts puzzle. This is not about finding the shortest way but about explaining different strategies!

B-Setti‘s rules – Advanced Examples

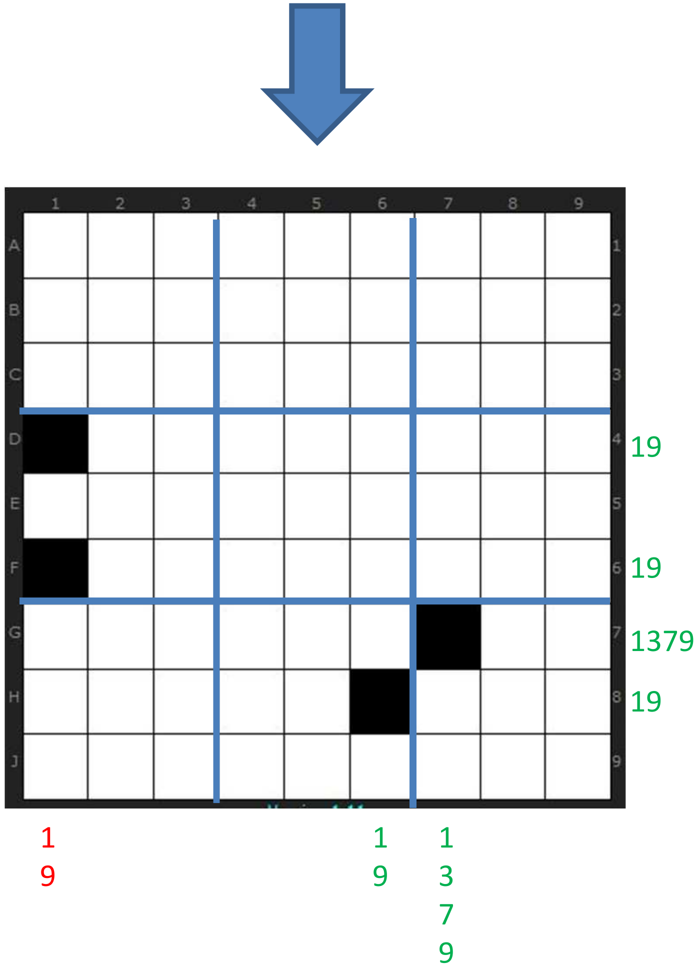


Look at box 4: box 4 contains two black cells, therefore it‘s missing exactly two digits. By **rule III** each of these digits must be missing from at least one of the rows D,E,F and one of the columns 1,2,3. The only digits for which this may be true are 1 and 9. Therefore 1 and 9 are missing in box 4.

Look at cols 1,2,3: Since 1 and 9 are missing from box 4 by **rule III** they have to be missing from one of the cols 1,2,3, so they are obviously missing from col 1.
Therefore 3 and 7 are in col 1.

Look at rows D,E,F: Since 1 and 9 are missing from box 4 they have to be missing from one of the rows D,E,F, so obviously one is missing from D, the other is missing from F.

B-Setti‘s rules – Advanced Examples

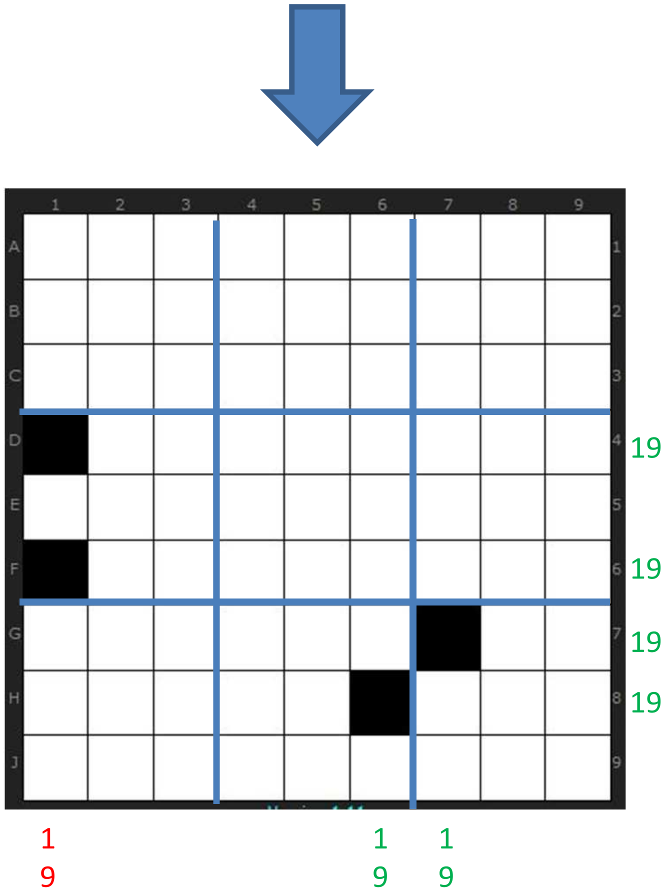


So far so good, but we can do more. We now use a combination of Setti‘s rule and our elimination strategies:

Look at box 9 and digit 3:
By the old Setti‘s rule 3 is either in row G and col 7 or it‘s missing in both.
If 3 is missing in col 7 then HJ7=12.
If 3 is missing in row G then G89=12.
Since HJ7 and G89 all belong to box 9 both conditions can‘t hold true at the same time.
Therefore 3 is in row G and col 7.

By a similar argument 7 is in row G and col 7.

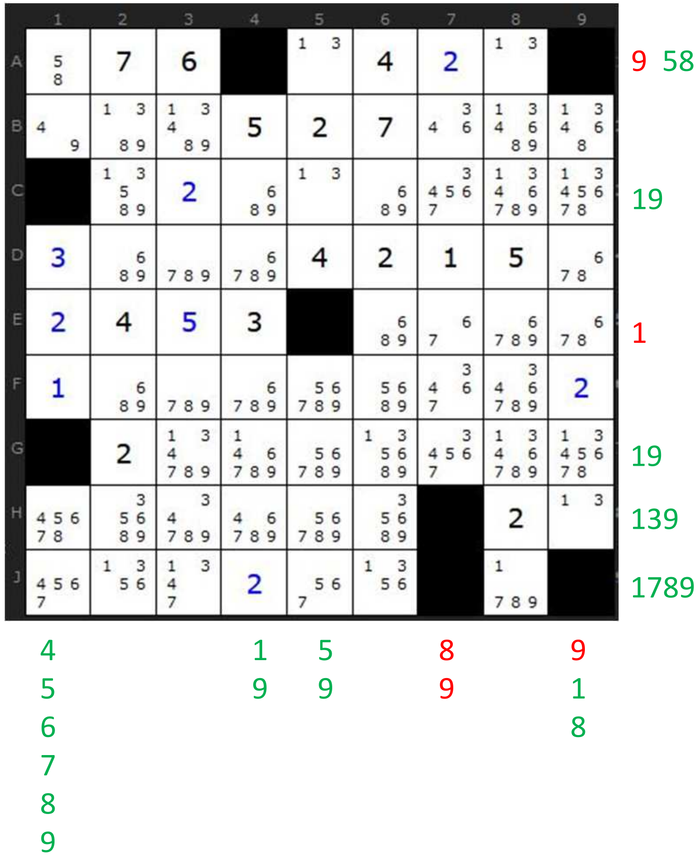
B-Setti‘s rules – Advanced Examples



So just by using B-Setti‘s rules and a little CE (careful elimination) we have really cleared up the lists of possible missing candidates!

B-Setti‘s rules – Advanced Examples

Example 23: is.gd/kst_680_B (*) after 8 solved cells

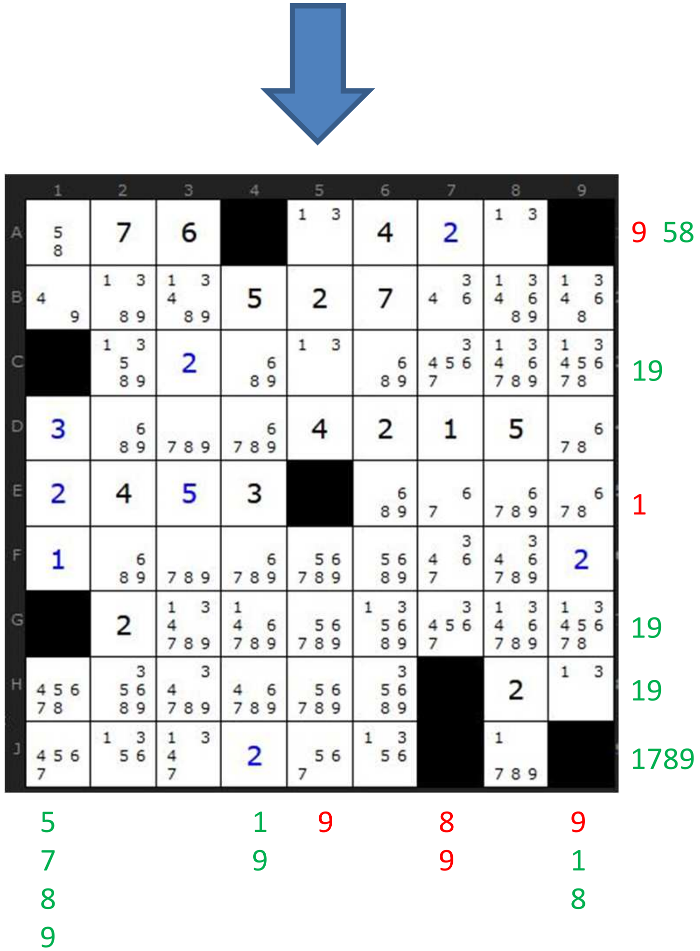


We again start with a BCA and write down all digits that must or might be missing in rows/columns.
Then we apply the normal Setti‘s rule:
4 and 6 in all rows => 4 and 6 in col 1.
3 in all cols => 3 in row H.

Now we apply B-Setti‘s-Rules:
Look at 5: 5 may be missing from cols 1,5 and from row A. Therefore 5 might be missing from boxes 1,2. But obviously 5 is present in box 2, therefore it might only be missing in box 1, therefore 5 must be in col 5.

(*) Posted by kst in week #680 on https://www.Str8ts.com/weekly_Str8ts.aspx.

B-Setti‘s rules – Advanced Examples

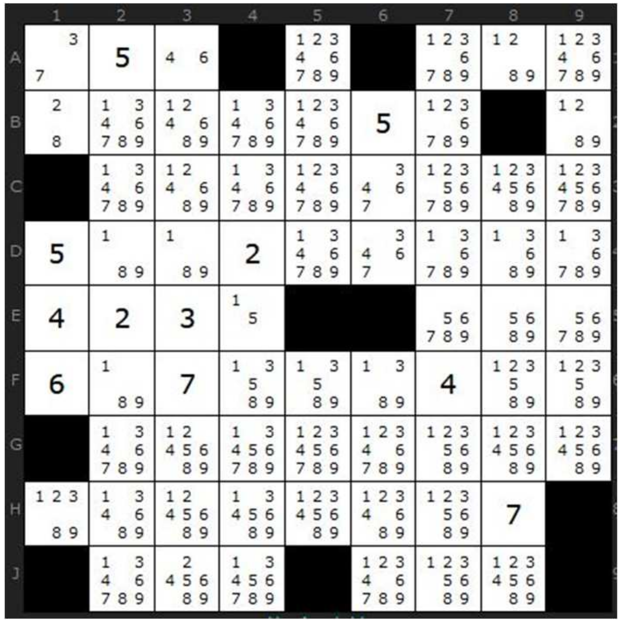


Look at box 9:
box 9 contains three black cells, therefore it‘s missing exactly three digits. By **rule III** each of these digits must be missing from at least one of the rows GHJ and one of the columns 7,8,9. The only digits for which this may be true are 1,8,9. Therefore 1,8,9 are missing in box 9.

$\Rightarrow$ J8=7, H9=3, G789=456 … and the rest of the puzzle is easily solvable.

B-Setti‘s rules – Advanced Examples

Example 24: is.gd/kst_692_B (*)after 3 solved cells.

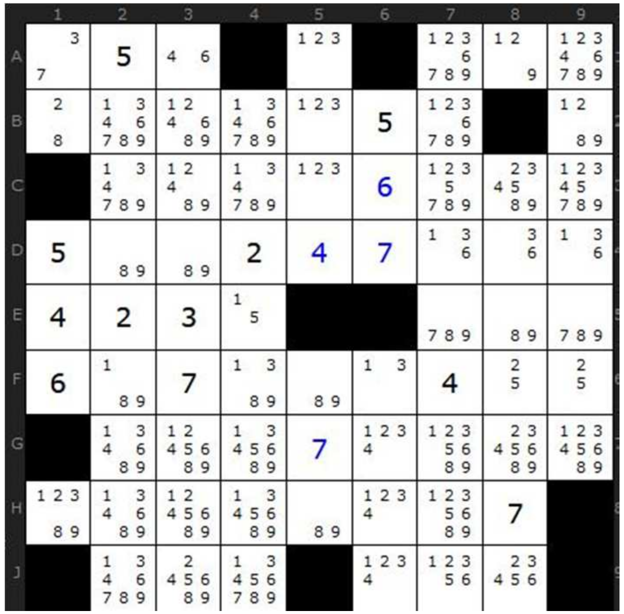


In this situation we do not do a full BCA, but just **look at rows D,E,F and box 5**: It‘s clear that the two digits missing from box 5 are exactly the two digits missing from E. So let‘s examine E:
2,3,4,7 are present, 1,5,6,8,9 might be missing.
Now, this is really helpful, because:
4 and 7 in E => 4,7 in box 5 => **hidden pair D56=47**.
This is a game changer, because now there‘s no cell left for 6 in box 5 and we can use **B-Setti‘s-rule III**:
6 not in box 5 => 6 not in E.
6 not in box 5 => 6 missing in one of cols 4,5,6
=> 6 missing in col 5.

Note: 6 missing in col 5 => D5=4, D6=7.
Applying this new found knowledge and cleaning up we get:

(*) Posted by kst in week #692 on https://www.Str8ts.com/weekly_Str8ts.aspx.

B-Setti‘s rules – Advanced Examples



Now **look at row E / box 5** again: It‘s clear that besides 6 either 1 or 5 is missing from E and therefore from box 5. So:
If E4=1 then no 5 in box 5.
If E4=5 then no 1 in box 5.
Because of this we can remove 1 and 5 from all other cells in box 5.
$\Rightarrow$ F6=3, F4=F5=89, ...

Note: Since just one of digits 1,5 is present in box 5 E4=15 is equivalent to a **large gap** in a „normal“ Str8ts puzzle.

B-Setti‘s rules – Proof

B-Setti‘s rule I: In the final solution of a B-Str8ts puzzle each digit occurs in the same number of columns and rows **and boxes**.
Proof: If a digit x occurs in exactly n rows (and therefore by the old Setti‘s rule in n columns) then by the Str8ts rules this digit is present in exactly n cells of the puzzle‘s solution. Since x may appear at most once in each box these n cells have to belong to n different boxes. Therefore x occurs in exactly n boxes.

Writing down a general mathematically correct proof for B-setti‘s rules II and III would require some extra terminology. To keep things simple, I‘ll prove each rule for just one row or column or box.

B-Setti‘s rule II: If a digit is missing from a row/column it has to be missing from at least one of the boxes intersecting this row/column.
Proof for row A: If digit x is missing from A then x occurs at most two times in rows A,B,C and therefore at most two times in boxes 1,2,3 (the boxes intersecting A,B,C and containing the same cells as A,B,C). Therefore x is missing in at least one of boxes 1,2,3.

B-Setti‘s rules – Proof

B-Setti‘s rule III: If a digit is missing from a box it has to be missing from at least one of the rows and at least one of the columns intersecting this box.
Proof for box 4: Let digit x be missing from box 4. Assume that x is present in all rows intersecting box 4, i.e. in D,E,F. Then x occurs three times in D,E,F and therefore three times in boxes 4,5,6. Since x may occur at most once in each box it has to be present in all boxes 4,5,6. Contradiction.
The proof for columns works by the same argument.

Conclusion

I hope this text was helpful to you. If you‘d like to share comments or constructive criticism please mail to BP.Str8ts@web.de.
Please note that I am not a native English speaker. So corrections/suggestions regarding the language are also very welcome.

Thanks for your attention!

Thanks to kst for comments and for allowing me to use some of his puzzles as examples in this text.

BP